Journal of Cosmology and
Astroparticle Physics

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Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

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Received January 7, 2019
Revised September 10, 2019
Accepted October 8, 2019
Published October 23, 2019

Abstract. We explore the scalar field obtained under the conformal transformation of the spacetime metric $g_{\mu\nu}$ from the Jordan frame to the Einstein frame in $f(R)$ gravity. This scalar field is the result of the modification in the gravitational part of the Einstein's general relativistic theory of gravity. For $f(R) = \frac{R^{1+\delta}}{R_c^\delta}$, we find the effective potential of the scalar field and calculate the mass of the scalar field particle "scalaron". It is shown that the mass of the scalaron depends upon the energy density of standard matter in the background (in solar system, $m_\phi \sim 10^{-16}\,\mathrm{eV}$). The interaction between standard matter and scalaron is weak in the high curvature regime. This linkage between the mass of the scalaron and the background leads to the physical effects of dark matter and is expected to reflect the anisotropic propagation of scalaron in moving baryonic matter fields as in merging clusters (Bullet cluster, the Abell 520 system, MACS etc.). Such scenario also satisfies the local gravity constraints of $f(R)$ gravity. We further calculate the equation of state of the scalar field in the action-angle variable formalism and show its distinct features as the dark matter and dark energy with respect to energy density of the scalar field at different values of the model parameter δ.

Keywords: dark matter theory, modified gravity

ArXiv ePrint: 1811.03964

https://doi.org/10.1088/1475-7516/2019/10/052

Contents

1 Introduction

The explanation of the late-time accelerated expansion of the universe and dark matter are the two major challenges of the present day cosmology. There are several observational evidences such as Supernovae type Ia, Baryon Acoustic Oscillations (BAO), Cosmic Microwave Background anisotropies, weak gravitational lensing etc., [1–4] which indicate that the present universe is in the phase of accelerated expansion. There are several approaches to explain the late-time accelerated expansion and dark matter both. The most suitable model to explain such acceleration is the Lambda Cold Dark Matter (ΛCDM) model, where Λ is the well known cosmological constant [5, 6]. However, the cosmological constant faces two serious problems (i) coincidence problem (ii) fine tuning problem. The fundamental nature of the cosmological constant is unknown. An explicit matter component with strange characteristics is also introduced to explain this phenomena. It is known as the dark energy [7–10]. There is still another approach called as modified gravity, wherein the late-time accelerated expansion is explained without using any explicit dark energy component [11–16]. In modified gravity models, we have $f(R)$ theory, Braneworld models, Gauss-Bonnet dark energy models etc., out of which $f(R)$ theory is one of the simplest modified gravity models, wherein the Ricci scalar R of the Einstein-Hilbert action is generalised into a function $f(R)$ of R. Besides the existence of the late-time accelerated expansion of the universe, the observations of rotation curves of galaxies [17, 18] and gravitational lensing indicate the presence of new matter often known as dark matter. Rather strangely, this matter does not experience the electromagnetic interaction, even though it has the gravitational interaction with the normal matter and radiation. The fundamental nature of the dark matter is still mysterious. There are some famous candidates of dark matter like weakly interacting massive particles (WIMPs) [19]. At the same time, there exist several alternative approaches to explain the effects of dark matter in modified gravity theories [20–23].

Here, we discuss the dark matter problem in $f(R)$ gravity. There are three approaches to derive the field equations in $f(R)$ models (i) metric formalism (ii) Palatini formalism and (iii) metric-affine formalism. We use the metric formalism in $f(R)$ gravity and dark matter is explained without considering any extra matter component. In particular, we consider $f(R) = \frac{R^{1+\delta}}{R_c^\delta}$ model to explain the geometrical effects of dark matter. Previously, dark matter and dark energy problems have been studied with scalar field in refs. [24, 25]. The oscillations of the scalar field have been shown to contribute to dark energy in refs. [22, 24, 26, 27]. We find out the relation between equation of state w and energy density ρ_ϕ of the scalar field for different model parameters.

Thus, the present paper is organised in five sections. Section 2 contains the basic field equations of the $f(R)$ gravity and the conformal transformation from the Jordan frame to the Einstein frame. In section 3, the chameleon mechanism is discussed in the framework of our model and the mass of the scalaron as the dark matter particle has been calculated showing its dependence on the background matter density. Section 4 contains the discussion about the equation of state of the scalar field in the formalism of action-angle variable for different model parameters. We conclude and discuss our results in section 5.

2 Field equations of $f(R)$ gravity and conformal transformation

We consider the 4-dimensional action of the $f(R)$ gravity model given by some general function $f(R)$ of the Ricci scalar as

$$\mathcal{A} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R) + \mathcal{A}_m(g_{\mu\nu}, \Psi_m)\,, \tag{2.1}$$

where $\kappa^2 = 8\pi G$ and $\mathcal{A}_m$ is the action of the matter part with matter field Ψ_m. We assume the spacetime as homogeneous, isotropic and spatially flat. It is given by the Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime as

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]\,, \tag{2.2}$$

where $a(t)$ is time dependent scale factor and the speed of light $c = 1$.

Here, we use the metric formalism in which connections $\Gamma^\alpha_{\beta\gamma}$ are defined in terms of the metric tensor $g_{\mu\nu}$. Varying the action (2.1) with respect to $g_{\mu\nu}$, we obtain the field equations given by

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\Box F(R) = \kappa^2 T_{\mu\nu}\,, \tag{2.3}$$

where $F(R) \equiv \frac{\partial f}{\partial R}$ and $T_{\mu\nu}$ is the energy-momentum tensor for matter. The trace of field equations (2.3) is given by

$$3\Box F(R) + F(R)R - 2f(R) = \kappa^2 T, \tag{2.4}$$

where $T = -\rho + 3p$. Here, ρ and p are the energy density and pressure of matter, respectively. The trace of the field equations shows that the Ricci scalar is dynamical if $f(R) \neq R$. We rewrite the action (2.1) in the form

$$\mathcal{A} = \int\sqrt{-g}\left(\frac{1}{2\kappa^2}F(R)R - U\right)d^4x + \mathcal{A}_m, \tag{2.5}$$

where

$$U = \frac{F(R)R - f(R)}{2\kappa^2}. \tag{2.6}$$

We switch over to the Einstein frame to see the real effects of dark matter in form of the scalar degree of freedom. It is possible to derive an action in the Einstein frame under the conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \tag{2.7}$$

where Ω^2 is the conformal factor and an overhead tilde denotes the quantities pertaining to the Einstein frame. The corresponding Ricci scalars in the two frames are mutually related as

$$R = \Omega^2 \left(\tilde{R} + 6\tilde{\Box}\omega - 6\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega \right), \tag{2.8}$$

where

$$\omega \equiv \ln\Omega, \partial_\mu\omega \equiv \frac{\partial\omega}{\partial\tilde{x}^\mu}, \tilde{\Box}\omega \equiv \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\omega). \tag{2.9}$$

Thus, the action (2.5) under the conformal transformation is transformed as [33]

$$\mathcal{A} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}F\Omega^{-2}(\tilde{R} + 6\tilde{\Box}\omega - 6\tilde{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega) - \Omega^{-4}U\right] + \mathcal{A}_m. \tag{2.10}$$

The linear action in $\tilde{R}$ can be written by choosing

$$\Omega^2 = F. \tag{2.11}$$

We consider a new scalar field ϕ defined by

$$\kappa\phi \equiv \sqrt{\frac{3}{2}}\ln F. \tag{2.12}$$

Using the relations (2.11) and (2.12), the action in the Einstein frame is found as

$$\mathcal{A} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + \mathcal{A}_m, \tag{2.13}$$

where

$$V(\phi) = \frac{U}{F^2} = \frac{FR - f}{2\kappa^2F^2} \tag{2.14}$$

stands as the potential term of the scalar degree of freedom in a general $f(R)$ model.

3 Dark matter like effects of scalaron in $f(R) = \frac{R^{1+\delta}}{R_c^\delta}$ type models

From the constant tangential velocity condition on the motion of a test particle in the stable orbits of the spiral galaxies, the form of $f(R)$ can be given by $f(R) \propto R^{1+\delta}$, where $\delta \ll 1$ is related to the tangential velocity [20]. Therefore, to solve the problem of dark matter, only very small deviation from general relativistic theory is required. However, we use a scalar field approach to overcome the dark matter issue in $f(R)$ gravity as obtained through the conformal transformation presented in the previous section. In this approach, it is shown that the dark matter can be scalar field particle "scalaron". In the Einstein frame the scalar field ϕ is coupled with non-relativistic matter. This coupling has the relation

$$\Omega^2 = F = e^{-2Q\kappa\phi}, \tag{3.1}$$

where Q is the strength of coupling. Now, from equations (2.12) and (3.1), Q is given by

$$Q = -\frac{1}{\sqrt{6}}. \tag{3.2}$$

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We consider the dark matter as the scalar field arising in the $f(R)$ model given by

$$f(R) = \frac{R^{1+\delta}}{R_c^{\delta}}\,, \tag{3.3}$$

where R_c is a constant having unit of the Ricci scalar R and δ is a small parameter of the model. Differentiating equation (3.3) with respect to R, we obtain

$$F = (1+\delta)\frac{R^{\delta}}{R_c^{\delta}}. \tag{3.4}$$

From equations (3.1) and (3.4) with $Q = -\frac{1}{\sqrt{6}}$, the Ricci scalar R in terms of scalar field ϕ is given as

$$R = R_c\left[\frac{e^{\sqrt{2/3}\kappa\phi}}{1+\delta}\right]^{\frac{1}{\delta}}. \tag{3.5}$$

The scalar field ϕ remains positive for $(1+\delta)R^{\delta} > R_c^{\delta}$. In case of small coupling $\kappa\phi \ll 1$, $\phi = \sqrt{6}\delta/2\kappa$, for $R = R_c$.

The variation of the action (2.13) with respect to ϕ yields the equation of motion of the scalar field given by

$$\tilde{\Box}\phi = V'(\phi) + \frac{\kappa}{\sqrt{6}}\tilde{T}\,, \tag{3.6}$$

where $\tilde{\Box}\phi = \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi)$, $V'(\phi) = \frac{dV}{d\phi}$ and $\tilde{T} = \tilde{g}^{\mu\nu}\tilde{T}_{\mu\nu}$. Since the traceless electromagnetic fields will not directly couple to scalaron, as also shown through the Lagrangian of the massless vector fields in the Einstein frame, the scalaron may exist as a dark matter particle. Equation (3.6) can also be written as

$$\tilde{\Box}\phi = V'_{\text{eff}}(\phi)\,, \tag{3.7}$$

where $V'_{\text{eff}}(\phi) = V'(\phi) + \frac{\kappa}{\sqrt{6}}\tilde{T}$. Using (2.14) with (3.3) and (3.4) for $V(\phi)$ and adding it to the contribution arising from matter, the effective potential of the scalaron for non-relativistic matter is given by

$$V_{\text{eff}}(\phi) = \frac{\delta R_c}{2\kappa^2(1+\delta)^{\frac{(1+\delta)}{\delta}}}e^{\sqrt{\frac{2}{3}}\frac{(1-\delta)}{\delta}\kappa\phi} + \frac{1}{4}\rho e^{\frac{-4\kappa\phi}{\sqrt{6}}}\,. \tag{3.8}$$

To find the value of the scalar field ϕ at which $V_{\text{eff}}(\phi)$ is minimum, we find out $\frac{dV_{\text{eff}}}{d\phi}$ given as

$$V'_{\text{eff}}(\phi) = \frac{R_c}{\sqrt{6}\kappa}\frac{(1-\delta)}{(1+\delta)^{\frac{1+\delta}{\delta}}}e^{\sqrt{\frac{2}{3}}\frac{(1-\delta)}{\delta}\kappa\phi} - \frac{\kappa}{\sqrt{6}}\rho e^{\frac{-4\kappa\phi}{\sqrt{6}}}\,. \tag{3.9}$$

Solving $V'_{\text{eff}}(\phi) = 0$, we obtain the value of ϕ at the minimum of $V_{\text{eff}}(\phi)$ given by,

$$\phi_{\text{min}} = \sqrt{\frac{3}{2}}\frac{1}{\kappa}\ln\left[(1+\delta)\left(\frac{\kappa^2\rho}{R_c(1-\delta)}\right)^{\frac{\delta}{1+\delta}}\right]. \tag{3.10}$$

For the calculation of the mass of the scalar field, we have

$$V''_{\text{eff}}(\phi) = \frac{R_c}{3}\frac{(1-\delta)^2}{\delta(1+\delta)^{\frac{1+\delta}{\delta}}}e^{\sqrt{\frac{2}{3}}\frac{(1-\delta)}{\delta}\kappa\phi} + \frac{2\kappa^2}{3}\rho e^{\frac{-4\kappa\phi}{\sqrt{6}}}\,, \tag{3.11}$$

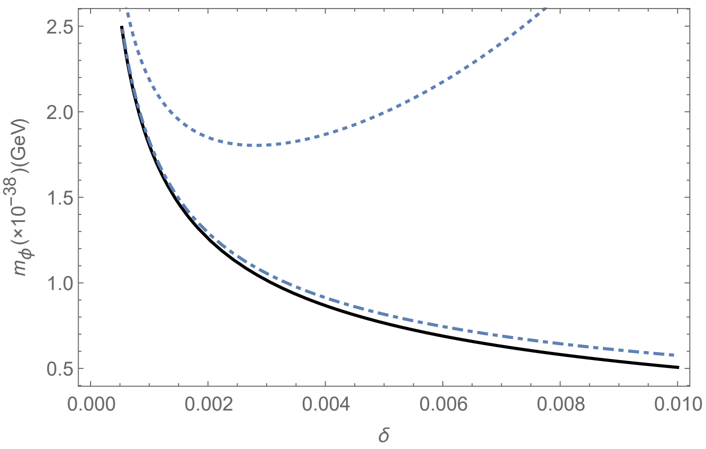

Figure 1. Plot for the variation of the scalaron mass m_ϕ with parameter δ. Here, the solid curve corresponds to $R_c = \Lambda$ (value of the cosmological constant), while the closely approximating dash-dotted curve, and the dotted curve (on the top) correspond to $R_c = 10^{-78}(\mathrm{GeV})^2$ and $R_c = 1(\mathrm{GeV})^2$, respectively. The value of the energy density of matter at the galactic scale is $\rho = 4 \times 10^{-42}(\mathrm{GeV})^4$ for all three curves.

which for the value of $\phi_{\min}$ (3.10) becomes

$$V''_{\mathrm{eff}}(\phi_{\min}) = \frac{(1-\delta)^{\frac{2\delta}{1+\delta}}}{3\delta(1+\delta)}(R_c)^{\frac{2\delta}{\delta+1}}(\kappa^2\rho)^{\frac{1-\delta}{1+\delta}}. \tag{3.12}$$

Thus, mass of the scalaron ($= V''_{\mathrm{eff}}(\phi_{\min})$) is clearly given by

$$m_\phi^2 = \frac{(1-\delta)^{\frac{2\delta}{1+\delta}}}{3\delta(1+\delta)}(R_c)^{\frac{2\delta}{\delta+1}}(\kappa^2\rho)^{\frac{1-\delta}{1+\delta}}. \tag{3.13}$$

It becomes clear that the mass of the scalar field depends upon the energy density of standard matter. Considering a very small $\delta \sim 10^{-6}$, at the electro-weak scales $\rho_{\mathrm{EW}} \sim (100\,\mathrm{GeV})^4$, $m_\phi \sim 10^{-3}$ eV, while at the solar system scales $\rho_\odot \sim 10^{19}\,\mathrm{eV}^4$, scalaron must be very light as $\sim 10^{-16}\,\mathrm{eV}$. Figure 1 shows the dependence of the scalaron mass on the model parameter δ over its very small values. When $R_c = \Lambda$ or even six order of magnitude higher, at $R_c = 10^{-78}(\mathrm{GeV})^2$, the scalaron mass decreases with increasing δ, while in case of $R_c = 1(\mathrm{GeV})^2$, m_ϕ decreases sharply initially for smaller values of δ and then increases for its larger values for the given energy density of matter ρ at the galactic scale. Since δ substantially determines the form of the $f(R)$ model, therefore, we have the model dependent mass of the scalaron. In all three cases, it rises to infinity as δ tends to zero and our model approaches the standard general relativistic description. From figure 2, it is obvious that the scalar field is coupled with the standard matter and the mass of the scalaron becomes large in the high curvature regions, although the coupling is weak around large curvature and the Compton wavelength of scalaron becomes small there. This provides a physical constraint on the scalaron mass such that it should be able to interact as a dark matter particle with the standard matter in its neighbourhood. However, the behaviour of scalar field changes with the length scale due to varying energy density of matter. We notice that even a small change of δ makes a large difference for such dependence of the mass on the energy density background. In particular, figure 3 shows that the behaviour of m_ϕ

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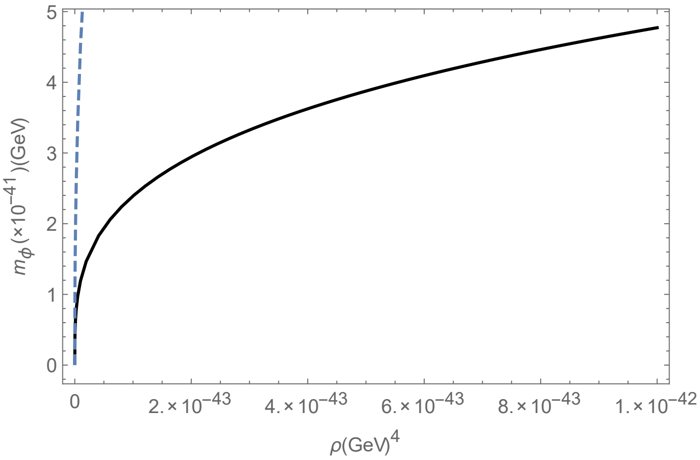

Figure 2. Plot for the variation of scalaron mass m_ϕ with the energy density ρ of matter. Here, solid and dashed curves correspond to $\delta = 0.25$ and $\delta = 0.10$, respectively, with $R_c = \Lambda$.

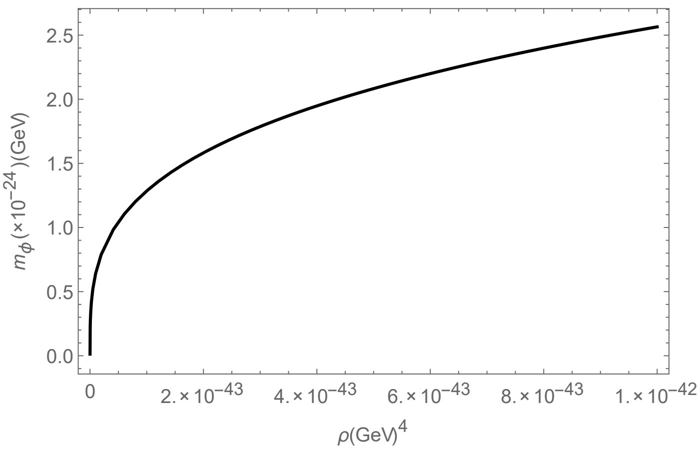

Figure 3. Plot for the variation of scalaron mass m_ϕ with the energy density ρ of matter corresponding to $\delta = 0.25$ and $R_c = 1(\text{GeV})^2$.

with respect to energy density ρ for $R_c = 1(\text{GeV})^2$ is reproduced as that for $R_c = \Lambda$, for the same $\delta = 0.25$ even though the mass m_ϕ now gets much higher. From the above figures, we find that the scalaron mass m_ϕ is large for smaller values of δ and high energy density of matter ρ. Since, the chameleon mechanism works in the high energy density regions, where the Compton wavelength of the scalaron becomes too small, therefore, the motion of the scalar field is diminished and screened out. It is consistent with the local gravity constraints. These properties of scalaron show that it could reproduce the effects of dark matter. The form $f(R) \propto R^{1+\delta}$, where δ is of the order of 10^{-6} as the squared tangential velocity of a test particle in the circular orbits around the galactic centres, is consistent with the local gravity constraints [20]. It is possible to constrain the mass of the scalaron using the energy density of matter from the large scale structure observations and δ from the galactic rotational velocities, with R_c as a scaling parameter. This mass can then be compared with the mass of the cold dark matter particle in the standard model, and also with the bounds from the decay widths of scalaron as a dark matter particle. It is also expected that the large

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translational velocities of the baryonic matter (such as of merging clusters) would induce anisotropy in the scalaron mass through the corresponding anisotropies in δ and R_c. This may cause a directional propagation of the scalar degree of freedom in the anisotropic, large curvature background. From the fact that δ, R_c and ρ determine the unique form of the effective potential, it is clear that we can find the exact viable form of the $f(R)$ model to reproduce the effects of dark matter in such high mass density regions.

4 Dark matter and the motion of the scalar field

The equation of motion of the scalar field (3.7) can be written as

$$\frac{d^2\phi}{d\tilde{t}^2} + 3\tilde{H}\frac{d\phi}{d\tilde{t}} + V'_{\rm eff}(\phi) = 0, \tag{4.1}$$

where $\tilde{H}$ is the Hubble parameter in the Einstein frame. When $\tilde{H} = 0$, the energy of the system is conserved and the oscillations are periodic. On the other hand, when $\tilde{H} \neq 0$, then it acts to produces a dissipative force against the oscillations of the scalar field. In this case, the energy of the field system is not conserved and the motion of the scalar field is not periodic. If the Hubble parameter $\tilde{H}$ varies slowly (adiabatically) with time during the time period T of the oscillation such that

$$\tilde{H} \ll \nu\,, \tag{4.2}$$

where ν is the frequency of the oscillations, then the rate of loss of energy, being proportional to the Hubble parameter, is very small and oscillations are approximately periodic.

Previously, several authors have studied the solution of the scalar field in the action-angle formalism to explain the dark energy [28, 29]. Motion of a test particle in $f(R)$ gravity has also been studied through the action angle variable formalism in our previous work [30]. We adopt this formalism in the present case to derive the equation of state of scalar field using the action-angle variable J which is defined as [31–33]

$$J = \oint p d\phi = 2\int_{\phi_1}^{\phi_2} \sqrt{2(\rho_\phi - V_{\rm eff}(\phi))} d\phi\,, \tag{4.3}$$

where p is the momentum and ρ_ϕ is the energy density of scalar field. ϕ_1 and ϕ_2 are the values of ϕ at which $V_{\rm eff}(\phi_1) = V_{\rm eff}(\phi_2) = \rho_\phi$.

The equation of state w of the scalar field is defined as

$$w = -1 + \frac{J}{\rho_\phi}\frac{1}{dJ/d\rho_\phi}. \tag{4.4}$$

We obtain the turning points ϕ_1 and ϕ_2, and J for the cases $\delta = 0.05, 0.10, 0.20$ and 10^{-6} to calculate w.

Figures 4, 5, 6 and 7 show the plots of the equation of state w of the scalar field with energy density ρ_ϕ of the scalar field. Here, the energy density of the non-relativistic matter is $4 \times 10^{-42} (\mathrm{GeV})^4$. It is found that w is negative, zero and positive in different regions of energy density ρ_ϕ. When $w = 0$, then the scalar field can behave as cold dark matter and when $w = \frac{1}{3}$, it corresponds to radiation. Negative equation of state i.e. $w < 0$ corresponds to the field with negative pressure. From figure 4, we find that initially, when energy density is small, equation of state is negative. In such case, with an upper bound, the scalar field

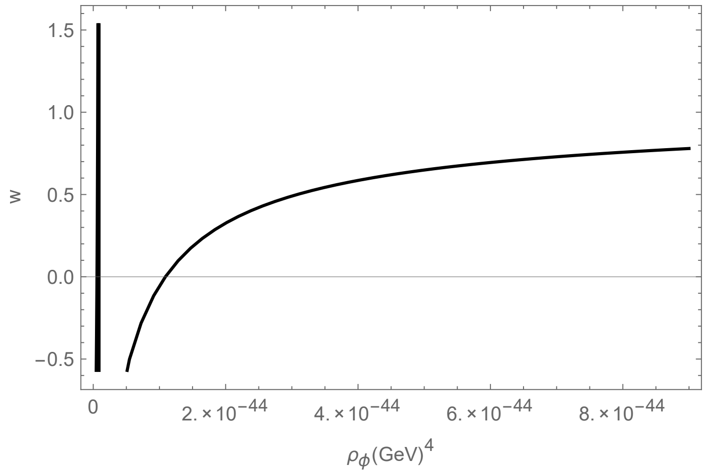

Figure 4. Plot for the variation of equation of state w with the energy density ρ_ϕ of scalar field corresponding to $\delta = 0.20$. It is found that the equation of state of the scalar field w is zero at about $\rho_\phi = 1 \times 10^{-44}(\text{GeV})^4$ and becomes positive at higher energy densities.

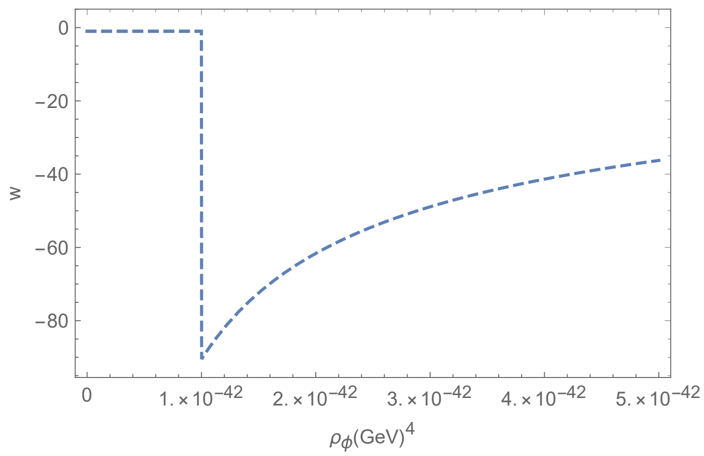

Figure 5. Plot for the variation of the equation of state of the scalar field w with the energy density ρ_ϕ of the scalar field corresponding to $\delta = 10^{-6}$.

behaves as dark energy. Further, when ρ_ϕ increases, equation of state rises to zero and it acts like cold dark matter. When ρ_ϕ increases further, equation of state w becomes positive.

Figure 5 shows the behaviour of equation of state w with energy density of the scalar field ρ_ϕ with $\delta = 10^{-6}$ i.e. a very small deviation from the general relativity. Here, $w = -1$, for a certain range of energy density ρ_ϕ of scalar field. At a certain value of energy density ρ_ϕ, equation of state w decreases drastically and after that w approaches to -1, when ρ_ϕ increases further.

The form of the effective potential around the ϕ_{min} is $\frac{1}{2}m_\phi^2\phi^2$ type and the action angle variable J is given by

$$J = \frac{2\pi\rho_\phi}{m_\phi}\,. \tag{4.5}$$

Now, using equation (4.4), we find that the equation of state w of the scalar field is zero which corresponds to that of the non-relativistic matter. Therefore, scalar field ϕ behaves

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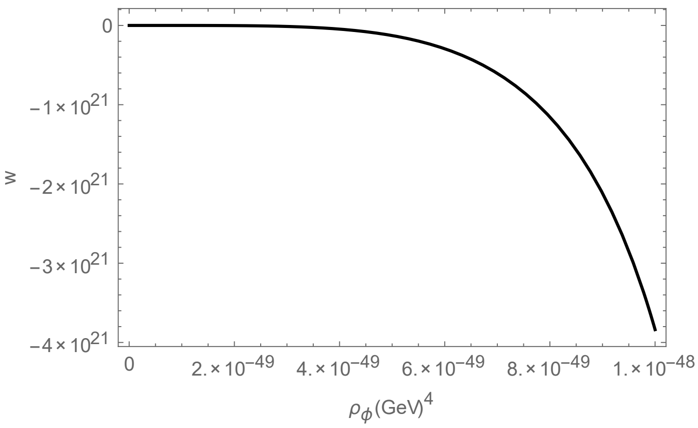

Figure 6. Plot for the variation of equation of state w with the energy density ρ_ϕ of scalar field corresponding to $\delta = 0.10$.

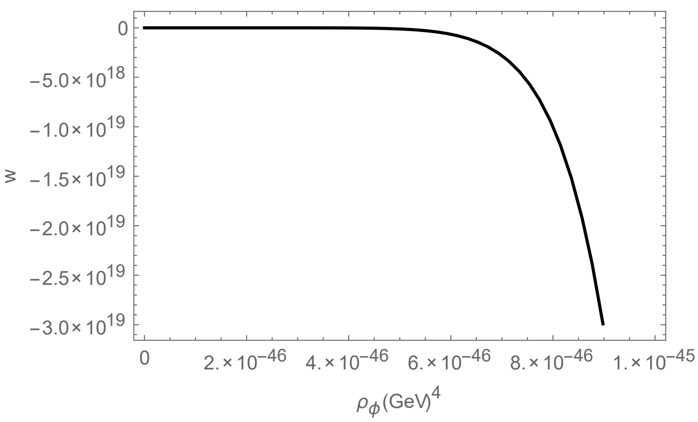

Figure 7. Plot for the variation of equation of state w with the energy density ρ_ϕ of scalar field corresponding to $\delta = 0.05$.

like Cold Dark Matter (CDM). If, the coupling between the scalar field ϕ and the normal matter is small and the universe is dominated by the scalar field, then the Hubble parameter $\tilde{H}$ in the Einstein frame is given by

$$\tilde{H}^2 = \frac{8\pi G}{3}\rho_\phi \, . \tag{4.6}$$

From equations (4.1) and (4.6), the energy density of the scalar field ϕ having the potential of the form $\frac{1}{2}m_\phi^2\phi^2$, varies as

$$\rho_\phi = \rho_\phi^{(0)} \left(\frac{\tilde{a}}{\tilde{a}_0}\right)^{-3} , \tag{4.7}$$

where $\rho_\phi^{(0)}$ and $\tilde{a}_0$ are the present energy density and the scale factor of the universe in the Einstein frame, respectively. In terms of the present density parameter $\Omega_\phi^{(0)}$ of the scalar field and redshift z, the energy density of the scalar field is given by

$$\rho_\phi = \rho_c^{(0)}(1+z)^3\Omega_\phi^{(0)} \, , \tag{4.8}$$

where $\rho_c^{(0)}$ is the present critical energy density $\left(\frac{3\tilde{H}_0^2}{8\pi G}\right)$. Since the scalar field ϕ behaves like Cold Dark Matter (CDM), therefore

$$\rho_{\rm CDM} = \rho_c^{(0)}(1+z)^3\Omega_{\rm CDM}^{(0)}\,. \tag{4.9}$$

In [34], authors considered the average over the single period of the harmonic oscillation of the scalar field ϕ as the energy density of the CDM, $\rho_{\rm CDM}$ and $V(\phi_{\rm min})$, the energy density of the dark energy $\rho_{\rm DE}$. In our case, we have

$$V_{\rm eff}(\phi_{\rm min}) = \frac{\delta R_c}{2\kappa^2}(1+\delta)^{\frac{2(1-\delta)}{\delta}}\left(\frac{\kappa^2\rho}{R_c(1-\delta)}\right)^{\frac{1-\delta}{1+\delta}} + \frac{\rho}{4(1+\delta)^2}\left(\frac{\kappa^2\rho}{R_c(1-\delta)}\right)^{\frac{-2\delta}{1+\delta}}\,. \tag{4.10}$$

From the observations, the relation between the present energy density of CDM, $\rho_{\rm CDM}^{(0)}$ and that of the dark energy $\rho_{\rm DE}^{(0)}$ is given by

$$\rho_{\rm CDM}^{(0)} \simeq \frac{3}{7}\rho_{\rm DE}^{(0)}. \tag{4.11}$$

If we consider the current effective potential at $\phi_{\rm min}$ as the present energy density of the dark energy, then

$$\rho_{\rm CDM}^{(0)} \simeq \frac{3}{7}V_{\rm eff}^{(0)}(\phi_{\rm min}) \equiv \frac{3}{7}\rho_{\rm DE}^{(0)}\,, \tag{4.12}$$

where $V_{\rm eff}^{(0)}(\phi_{\rm min})$ is the present value of $V_{\rm eff}(\phi_{\rm min})$.

Therefore, from equations (4.9), (4.10) and (4.12) at the present epoch ($z=0$), we have

$$\Omega_{\rm CDM}^{(0)} = \frac{3}{7\rho_c^{(0)}}\left[\frac{\delta R_c}{2\kappa^2}(1+\delta)^{\frac{2(1-\delta)}{\delta}}\left(\frac{\kappa^2\rho^{(0)}}{R_c(1-\delta)}\right)^{\frac{1-\delta}{1+\delta}}\right] + \frac{3}{7\rho_c^{(0)}}\left[\frac{\rho^{(0)}}{4(1+\delta)^2}\left(\frac{\kappa^2\rho^{(0)}}{R_c(1-\delta)}\right)^{\frac{-2\delta}{1+\delta}}\right], \tag{4.13}$$

where $\rho^{(0)}$ corresponds to the present value of the energy density of the standard matter.

Now, using the observational values of $\rho_c^{(0)} = 1.38\times10^{-48}(\text{GeV})^4$, $R_c = 1.0\times10^{-83}(\text{GeV})^2$ and $\rho^{(0)} = 8.4\times10^{-49}(\text{GeV})^4$ from the Planck data [35] for $\delta = 0.10$ in equation (4.13), we obtain $\Omega_{\rm CDM}^{(0)} \approx 0.27$.

The equation (4.11) shows the relation between the present energy densities of the dark matter and dark energy. For any other epoch, such relation will be different. Therefore, as an initial condition in equation (4.13), we have used the present value of the energy density of the matter at the galactic scale ($\rho^{(0)} = 8.4\times10^{-49}(\text{GeV})^4$). Now, to calculate the energy density of the CDM at any other epoch such as the recombination epoch, we use equation (4.9) along with the value of $\Omega_{\rm CDM}^{(0)}$ calculated in equation (4.13). For instance, we calculate the value of the $\rho_{\rm CDM}$ at recombination epoch corresponding to $z = 1000$.

Substituting the values of $\rho_c^{(0)}$, $\Omega_{\rm CDM}^{(0)} = 0.27$ as calculated from the equation (4.13) and $z = 1000$ in equation (4.9), we find the value of energy density of the CDM at recombination epoch, $\rho_{\rm CDM} \approx 3.7\times10^{-40}(\text{GeV})^4$. Once $\Omega_{\rm CDM}^{(0)}$ is fixed then it will give the required value of the energy density of the dark matter for other epochs. Equation (4.9) shows that in our model one can find $\rho_{\rm CDM}$ at any arbitrary epoch in past.

Thus, it is found that the value of $\Omega_{\rm CDM}^{(0)}$ obtained is consistent with the observations. For $z = 0$ corresponding to the present epoch, we determine $\Omega_{\rm CDM}^{(0)} = 0.27$. In figures 8 and 9,

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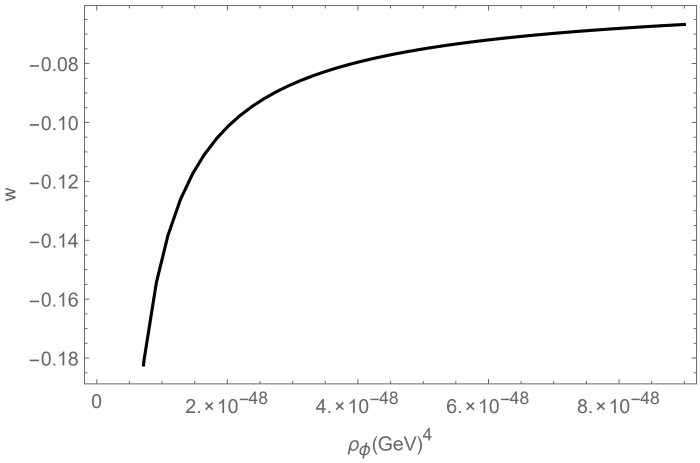

Figure 8. Plot for the variation of equation of state w with the energy density ρ_ϕ of scalar field corresponding to $\delta = 0.10$. It corresponds to the present epoch at $z = 0$ and $\rho = 8.4 \times 10^{-49}(\mathrm{GeV})^4$.

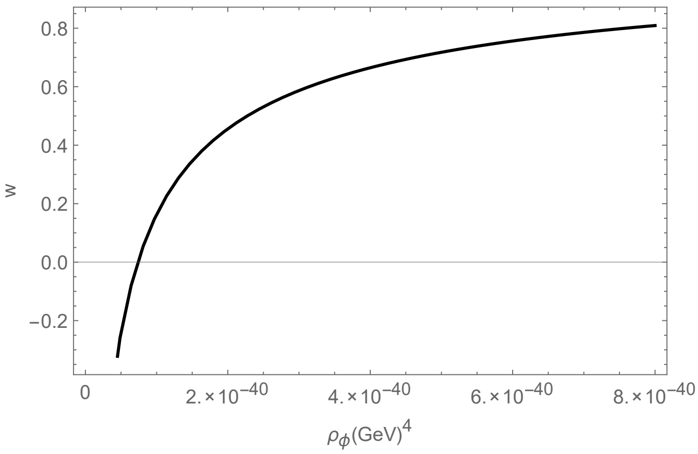

Figure 9. Plot for the variation of equation of state w with the energy density ρ_ϕ of scalar field corresponding to $\delta = 0.10$. It corresponds to the recombination epoch at $z = 1000$ and $\rho = 4 \times 10^{-39}(\mathrm{GeV})^4$.

we have used $\delta = 0.10$ in each case and the normal matter densities $\rho = 8.4 \times 10^{-49}(\mathrm{GeV})^4$ and $\rho = 4 \times 10^{-39}(\mathrm{GeV})^4$ obtained from the observations for present and recombination epochs, respectively. In figures 8 and 9, we have $w \simeq 0$ corresponding to the energy densities of the normal matter for these epochs. It shows that $f(R) = \frac{R^{1+\delta}}{R_c^\delta}$ for $\delta = 0.10$ can viably act like dark matter.

5 Summary and discussion

We investigated here the possibility of explaining the dark matter problem in the framework of $f(R)$ gravity. In the Einstein frame, scalaron appears as the dark matter particle that does not directly couple to the traceless electromagnetic fields and remains dark with respect to such interaction. Further, being a fluctuation in a background of scalar field ϕ_{min} where $V_{\mathrm{eff}}(\phi)$ is minimum, this does not need to add any new matter component to the particle inventory of the standard model. However, the effective scalaron potential is influenced by the energy density of the standard matter in the background. In our model, with $f(R) = \frac{R^{1+\delta}}{R_c^{\delta}}$, the mass of the scalaron almost linearly depends upon the energy density of standard matter. Therefore, it does not vary much in large curvature regions in comparison to the other models (like the Starobinsky model [36]) in which there is a power law dependence. This behaviour is suitable for the dark matter phenomena under screening conditions. In our model, the mass of the scalaron can be constrained in two ways using particle physics as well as the cosmological observations. One way is to find the decay widths of the scalaron and check the bounds on its mass. The other way is to obtain it directly from the observational data of the background matter energy density and the model parameters δ and R_c (from the galactic rotation and lensing etc.). This mass must then also be compared with the theoretical constraints obtained in the ΛCDM model. We have not found any conditions on the amount of the total scalaron content, or its time evolution, or the scale dependence of the δ and R_c i.e. the deviation from the Einstein's general relativistic theory at different scales, especially in the background of violent mergers of clusters. In such cases, the role of δ and R_c both is unclear, and may profoundly indicate a solution with anisotropic propagation of scalaron in response to the direction and magnitude of velocities of standard baryonic matter. This approach may lead to solve the puzzles of the offset of dark matter from the intra-cluster baryonic hot gases, as marked in the Bullet cluster (1E0657-56) [37], Abell 520 system [38], MACS [39] etc. Clearly, lensing must be a good tool to investigate such scale dependence and anisotropic propagation of the scalar degree of freedom in the large curvature background, with additional tools to check whether the large scale (low curvature) behaviour of the scalaron may produce the effects of dark energy causing acceleration in recent epochs. In general, it seems unlikely that δ and R_c must remain constant and isotropic over all scales and also throughout the time evolution of the universe. In view of the action-angle formalism, the variation of equation of state of the field with its energy density opens a way to connect the dark energy to dark matter in a single perspective. We will attempt to study these broader issues in our future work.

Acknowledgments

MMV thanks the Kavli IPMU, Tokyo, especially Hitoshi Murayama for hosting the visit, and Misao Sasaki and Masahiro Takada for useful discussions. Both authors thank IUCAA, Pune for the facilities where a part of the present work was completed under the associateship programme.

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