Fuzzy Optimization of Sustainable Production Model with Screening and Carbon Awareness for Manufacturing Industry using GMI method

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Abstract—**Environmental concerns have led to remanufacturing and repurposing rules in the industrial sector, with carbon emission being a significant concern. Implementing a carbon tax on manufacturing can help reduce emissions. Screening is crucial for improving product quality and is an impactful framework for the industry. The study establishes the traditional supply chain model, focusing on carbon emission cost and screening techniques for production. Remanufacturing and recycling of used products are effective decisions for promoting a greener environment and sustainable techniques. An illustration and sensitivity analysis are presented to validate the model stability.**

Keywords-:Carbon emission; Defective items; Graded Mean Integration method (GMI); Optimization; Sustainable Production; Screening, Trapezoidal fuzzy number;

Introduction

Product quality and delivery are almost equally important in the manufacturing industry. High-quality products can deliver high profits to the manufacturer, by improving the value of products. Reducing reworks and scrap rates, as well as decreasing maintenance costs and social losses caused by maintenance costs Environmental pollution and economics were the two main focuses of this study on sustainable management. Because of the outstanding contributions that several scholars have made over the past couple of decades, the concept of industrial sustainable development has gained much attention. Any company's ambition to meet financial goals and manage the waste that enters society may be considered sustainability. Reducing costs and preserving systemic profit is the major goal of the sustainable distribution network. The number of industries and a steady growth in net trash have expanded. [1] Proposed an economic order quantity (EOQ) framework with fuzzy demand and carbon tax. Furthermore, the use of green technology in manufacturing companies typically decreases emissions as technology develops and matures. A two-layer green distribution network for imperfect production has been proposed by [2] under the trade credit scheme, in which the rate of defectiveness has been taken as the function of manufacturing rate and time. For the meat production distribution network,[3] developed a three-stage linear programming model that can decide on the number of facilities and settlement sites. A multi-objective optimization model [4] is formed to identify the optimal number of inspectors at each level of expertise to satisfy the criteria of a screening center. Three-tiered sustainable procurement architecture is being proposed [5], with a single supplier, one manufacturer, and numerous retailers. [6] Investigated the government's participation in a three-tier distribution network that comprised one manufacturer and one retailer. Study of Supply chain models with imperfect quality items when end demand is sensitive to price and marketing expenditure by [7].

In order to reflect and represent assessment information in several dimensions and to communicate imprecise, incomplete, and inconsistent information while handling multi-criteria decision-making issues, one can use the Trapezoidal fuzzy Fuzzy Number, a more generalized platform. Fuzzy set theory is a mathematical model that builds on classical set theory to handle uncertainty and ambiguity in element membership. Zadeh [8] first introduced it in 1965. One of these generalizations, proposed by Atanassov [9] in 1984, is intuitionistic fuzzy set theory. Each element in an intuitionistic fuzzy set is given a membership function and a non-membership function, ensuring their total falls between zero and one. Thus, unlike Atanossov's [10] fuzzy sets, intuitionistic fuzzy sets can represent hesitancy degrees, which signify a lack of confidence or doubt in an element's membership and non-membership functions. Mondal and Roy [11] use intuitionistic fuzzy differential equations in a variety of domains, including oil production while Mondal and Roy [12] use them for weight loss issues. A sustainable inventory model for defective items under fuzzy environment [13].

Given the amount of carbon generated during production [14] described the remanufacturing inventory model using cap and trade regulation. [15] described a fuzzy optimization production model with a credit scheme. [16] describe the impact of carbon emissions with the trade credit scheme [17] studied the fuzzy optimization for nonlinear holding costs and stock-dependent demand in an economic order quantity model with the trapezoidal fuzzy number and graded mean integration method. [18] Investigate a fuzzy production inventory model with carbon emission using the sign distance method

Preliminaries

Trapezoidal fuzzy number: Trapezoidal fuzzy number is defined as the fuzzy set (on the set of real numbers), if the membership function of $\check{C} = (C_1, C_2, C_3, C_4)$ is as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - C_1}{C_1 - C_2}, & C_1 < x < C_2 \\ 1, & C_2 < x < C_3 \\ \dfrac{C_4 - x}{C_4 - C_3}, & C_3 < x < C_4 \\ 0, & \text{otherwise} \end{cases}$$

Graded Mean Integration method (GMI)- Graded Mean Integration method has been introduced by S.H. Chen and C.H. Hsieh in 1998. Let $\check{C} = (C_1, C_2, C_3, C_4)$ be a trapezoidal fuzzy number, then the GMI representation of $\check{C}$ is defined as

$$G(\check{C}) = \frac{\frac{1}{2}\int_0^1 \alpha\left[CL_{(\alpha)} + CR_{(\alpha)}\right]d\alpha}{\int_0^1 \alpha d\alpha} = \frac{1}{6}(C_1 + 2C_2 + 2C_3 + C_4)$$

Assumption and Notations

Assumption

Throughout the developed model, the following assumptions and notations are implemented.

- The time horizon T is supposed to be known and finite and lead time is zero.
- Generally, demand is assumed to remain constant in most cases. However, the demand is linear as $a + bt$ in this model to make it more realistic.
- It is considered that shortage is not permitted.
- Sustainable inventory system contains a single type of product.
- To maintain the reputation of industry, the idea of screening process is considered to assure excellent product-quality.

Notations

$S_u(t)$ =Ready to use stock of the usable items at time t
$S_{nu}(t)$ =Ready to use stock of the non-usable items at time t
M =The total number of cycles in T
U =The number of cycles in which useful goods can be purchased
C_f =other cycle's time duration in days.
P_1 =Manufacturing rate of the first cycle (Constant) in $
P_2 =Remanufacturing rate in $
P_3 =Other cycles manufacturing rate in $
$\widetilde{R_1}$ =Carrying cost of usable items in $/unit time
$\widetilde{R_2}$ =Carrying expense of non-usable products in $/unit time
$\widetilde{R_3}$ =Set up expense in $
$\widetilde{R_4}$ =Usable item's purchasing cost in $ /unit from market
$\widetilde{R_5}$ =Manufacturing cost in $ / unit
R_6 =Carbon emission tax in $
$\widetilde{R_7}$ =Remanufacturing cost in $
R_8 =Screening cost in $
d =Time duration of first cycle
d_m =Manufacturing time duration of first cycle
O_m =Manufacturing time duration of other cycles
S =Selling price in $
D =Disposal rate is constant
δ =Defective units produced during manufacturing
t_k =The time period in the k[th] cycle when useable goods are gathered for remanufacturing.

Mathematical formulation

The mathematical formulation was established with the purpose of maximizing the total profit of the system while considering the cost of carbon emissions and the screening process. In this research, considering an imperfect production system including recycling and remanufacturing of defective items and used items. While developing this model it is assumed that there are M manufacturing loops throughout the finite period horizon T. Remanufacturing begins in the second loop and it stays until the end loop. The demand of first loop is fulfilled only by high quality items from manufacturing and other cycle consumption is addressed by high quality items from both manufacturing and remanufacturing. Remanufactured products include non-serviceable commodities retrieved from the market as well as defective manufacturing. However, certain acquired non-serviceable goods that are not eligible for remanufacturing are removed from the circulation before remanufacturing may take place. Some acquired non-serviceable products that are not suitable for remanufacturing are disposed before remanufacturing. The level of serviceable and non-serviceable commodities is represented in Figure 1 and 2.

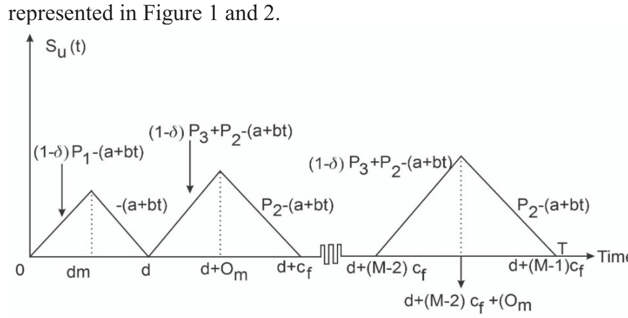

Fig.1. Stock level for serviceable goods

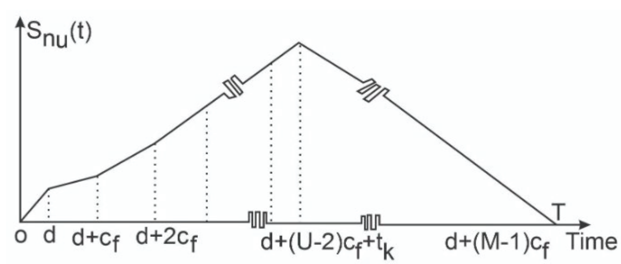

Fig.2. Stock level for non-serviceable goods

Let us consider the production–recycling material flow model. In this model, in the first cycle, P_1 represents the constant manufacturing rate. In other cycles, P_3 denotes the manufacturing rate, which is dependent on the remanufacturing rate. $S_u(t)$ represents the operable (serviceable) item's on-hand stock at time t. non-serviceable products $S_{nu}(t)$, which have been used products retrieved from the market and faulty manufacturing, are remanufactured. A certain amount of gathered non-serviceable goods that cannot recycled are disposed at the rate D before remanufacturing.

Mathematical modeling

4.1 Formulation for the first cycle

The mathematical expression explaining the usable stock level $S_u(t)$ in the period $0 < t < T$ is provided below,

$$\frac{dS_u(t)}{dt} = (1-\delta)P_1 - (a+bt), \qquad 0 \le t \le d_m \qquad (1)$$

$$\frac{dS_u(t)}{dt} = -(a+bt), \qquad d_m \le t \le d \qquad (2)$$

With the boundary situations $S_u(t) = 0$ at $t = 0$ and, $t = d$ where $\delta > 0$.

The mathematical expression describing the non-serviceable stock level $S_{nu}(t)$ in the period $0 \le t \le d$ is provided below,

$$\frac{dS_{nu}(t)}{dt} = \beta_{11}(a+bt) + \delta P_1 - D, \qquad (0 < \beta_{11} < 1), 0 \le t \le d \qquad (3)$$

The outcome of the equations (1) and (2) is given by,

$$S_u(t) = \begin{cases} (1-\delta)P_1 t - \left(a + \frac{bt}{2}\right)t & 0 \le t \le d_m \\ \left(a + \frac{bT}{2}\right)d - \left(a + \frac{bd_m}{2}\right)d_m & d_m \le t \le d \end{cases} \qquad (4)$$

The outcome of the equation (3) is as follows,

$$S_{nu}(t) = \left(\beta_{11}\left(a + \frac{bd}{2}\right) + \delta P_1 - D\right)d, \qquad 0 \le t \le d \qquad (5)$$

From equation (4),

$$d_m = \frac{\left(a + \frac{bd}{2}\right)}{(1-\delta)P_1}d, \qquad (6)$$

4.1.1. Carrying cost for the serviceable goods

Carrying expense is incurred when the stock is carried by the system for an extended period. Therefore, the carrying cost for the serviceable products is as follows,

$$RU_1 = \widetilde{R_1}\int_0^{d_m} S_u(t)dt + \widetilde{R_1}\int_{d_m}^{d} S_u(t)dt$$

$$= \widetilde{R_1}\left[\left\{(1-\delta)P_1 - \left(a + \frac{bd_m}{3}\right)\right\}\frac{(d_m)^2}{2}\right] + \widetilde{R_1}\left[\frac{b(d)^3}{2} + \frac{b(d_m)^3}{2} - \frac{b(d_m)^2 d}{2} - \frac{b(d)^2 d_m}{2}\right]$$

4.1.2. Carrying cost for the non-serviceable goods

The carrying expense for the non-serviceable goods is as follows,

$$RN_1 = \widetilde{R_2}\int_0^{c_f} S_{nu}(t)dt = \widetilde{R_2}\left\{\beta_{11}\left(a + \frac{bc_f}{2}\right)(c_f)^2 + (\delta P_1 - D)c_f\right\}$$

Selling price = $S\left[a(d-d_m) + \frac{b}{2}((d)^2 - (d_m)^2)\right]$

4.1.3. Manufacturing cost

Some manufacturing or material expenditures are also required to make the product. To do so, a manufacturing cost is estimated, which is as follows: $= \widetilde{R_5}P_1 d_m$

4.1.4. Ordering cost

The ordering cost can be estimated as follows: $= \widetilde{R_3}$

4.1.5. Purchasing cost

To purchase the material for manufacturing, a cost is utilized, which is known as purchasing cost. Purchasing cost can be evaluated as follows:

$$= \widetilde{R_4}\left[\beta_{11}\left\{a(d-d_m) + \frac{b}{2}((d)^2 - (d_m)^2)\right\}\right]$$

4.1.6. The overall profit in the first cycle

The overall profit in the first cycle is as follows,

$$\widetilde{TPC_1} = S\left[a(d-d_m) + \frac{b}{2}((d)^2 - (d_m)^2)\right] - \widetilde{R_1}\left[\left\{(1-\delta)P_1 - \left(a + \frac{bd_m}{3}\right)\right\}\frac{(d_m)^2}{2}\right] - \widetilde{R_1}\left[\frac{b(d)^3}{2} + \frac{b(d_m)^3}{2} - \frac{b(d_m)^2 d}{2} - \frac{b(d)^2 d_m}{2}\right] - \widetilde{R_2}\left\{\beta_{11}\left(a + \frac{bc_f}{2}\right)(c_f)^2 + (\delta P_1 - D)c_f\right\} - \widetilde{R_4}\left[\beta_{11}\left\{a(d-d_m) + \frac{b}{2}((d)^2 - (d_m)^2)\right\}\right] - \widetilde{R_5}P_1 d_m \qquad (7)$$

4.2 Mathematical expression for i[th] cycle $2 \le i \le M$

The differential equation defining the serviceable stock levels $S_u(t)$ in the period $\left(d + (i-2)c_f \le t \le d + (i-1)c_f\right)$ is defined as follows:

$$\frac{dS_u(t)}{dt} = (1-\delta)P_3 + P_2 - (a+bt), \quad d + (i-2)c_f \le t \le d + (i-1)c_f + O_m \qquad (8)$$

$$\frac{dS_u(t)}{dt} = P_2 - (a+bt), \qquad d + (i-2)c_f + O_m \le t \le d + (i-1)c_f \qquad (9)$$

With the boundary situations
$S_u(t) = 0 \; at \; t = d + (i-2)c_f$ and $t = d + (i-1)c_f$, where $P_2 < (a+bt)$.

Now, the differential equation defining the non-serviceable stock levels $S_{nu}(t)$ in the period $\left(d + (i-2)c_f \le t \le d + (i-1)c_f\right)$ is defined as,

$$\frac{dS_{nu}(t)}{dt} = \sum_{f=1}^{i}\beta_{if}(a+bt) + \delta P_3 - P_2 - D, \quad d + (i-2)c_f \le t \le d + (i-1)c_f \qquad (10)$$

$i \ge f, \quad i = 2,3, \dots \dots \dots U-1$

$$\frac{dS_{nu}(t)}{dt} = \sum_{f=1}^{U}\beta_{if}(a+bt) + \delta P_3 - P_2 - D, \quad d + (U-2)c_f \le t \le d + (U-1)c_f + t_k \qquad (11)$$

$$\frac{dS_{nu}(t)}{dt} = \delta P_3 - P_2, d + (U-2)c_f + t_k \le t \le d + (M-1)c_f \qquad (12)$$

With the boundary situations
$S_{nu}(t) = 0 \; at \; t = d + (M-1)c_f$, where $\sum_{f=1}^{i}\beta_{if} = i\beta_{11} + \frac{i(i-1)}{2}g \le 1$, for every $i = 2,3, \dots \dots U-1$

By equation (11) and (12),

$$t_k = \frac{(D-a)+\sqrt{(a-D)^2+2b}}{b}\left[\begin{array}{c}\{(P_2-\delta P_3)(M-U)c_f-(\delta P_3-P_2)c_f\} \\ -\left(\beta_{11}\left(a+\frac{bd}{2}\right)+\delta P_1-D\right)d \\ -\sum_{p=1}^{U-2}\left\{p\beta_{11}+\frac{p(p-1)}{2}g\right\}\left(a+\frac{bc_f}{2}\right)c_f \\ +U-3(P_2-\delta P_3)c_f+Dc_f \\ -\left\{\begin{array}{c}\left((U-1)\beta_{11}+\frac{(U-1)(U-2)}{2}g\right)\left(a+\frac{bc_f}{2}\right) \\ -D-(P_2-\delta P_3)\end{array}\right\}c_f\end{array}\right] \quad (13)$$

The outcome of the equations (8) and (9) as follows,

$$S_u(t)=\left\{\left((1-\delta)P_3+P_2-\left(a+\frac{bc_f}{2}\right)\right)O_mP_2\left(O_m-c_f\right)-\left\{a\left(O_m-c_f\right)+\frac{b\left((O_m)^2-(c_f)^2\right)}{2}\right\}\right\} \quad (14)$$

$$O_m=-\frac{\left(P_2-\left(a+\frac{bc_f}{2}\right)\right)}{(1-\delta)P_3}c_f \quad \text{Where } c_f=\frac{T-d}{M-1} \quad (15)$$

Thus, the entire cost's worth is made up of the following components.

Carrying cost for the usable products is defined as,

$$CU_i=\widetilde{R_1}\int_{d+(i-2)c_f}^{d+(i-2)c_f+O_m}S_u(t)dt+\widetilde{R_1}\int_{d+(i-2)c_f+O_m}^{d+(i-1)c_f}S_u(t)dt$$

$$=\widetilde{R_1}\left[\left\{(1-\delta)P_3+P_2-\left(a+\frac{bO_m}{2}\right)\right\}(O_m)^2+\left\{P_2-\left(a+\frac{b(O_m-c_f)}{2}\right)\right\}\left(2O_mc_f-(O_m)^2-(c_f)^2\right)\right] \quad (16)$$

Carrying cost for the non-usable products is defined as,

$$CN_2=\widetilde{R_2}\int_d^{d+c_f}S_{nu}(t)dt$$

$$=\widetilde{R_2}\left[\left\{(2\beta_{11}+g)\left(a+\frac{bc_f}{2}\right)-D-(P_2-\delta P_3)\right\}c_f+\left\{\beta_{11}\left(a+\frac{bc_f}{2}\right)+\delta P_1-D\right\}dc_f\right] \quad (17)$$

Carrying cost for non-usable products for i[th] cycle$(3\le i\le M-1)$ is as follows,

$$CN_i=\widetilde{R_2}\int_{d+(i-2)c_f}^{d+(i-1)c_f}S_{nu}(t)dt$$

$$=\widetilde{R_2}\left[\begin{array}{c}\left\{\left(\beta_{11}\left(a+\frac{bd}{2}\right)+\delta P_1-D\right)dt_k\right\} \\ - \\ +\sum_{p=2}^{i-1}\left\{p\beta_{11}+\frac{p(p-1)}{2}g\right\}\left(a+\frac{bc_f}{2}\right)(c_f)^2 \\ -(i-2)(P_2-\delta P_3)(c_f)^2-D(c_f)^2 \\ +\left\{\begin{array}{c}\left(i\beta_{11}+\frac{i(i-1)}{2}g\right)\left(a+\frac{bc_f}{2}\right) \\ -D-(P_2-\delta P_3)\end{array}\right\}c_f\end{array}\right] \quad (18)$$

Carrying cost for non-usable products for J[th] cycle is as follows,

$$CN_j=\widetilde{R_2}\int_{d+(U-2)c_f}^{d+(U-2)c_f+t_k}S_{nu}(t)dt+\widetilde{R_2}\int_{d+(U-2)c_f+t_k}^{d+(U-1)c_f}S_{nu}(t)dt$$

$$=\widetilde{R_2}\left[\begin{array}{c}\left\{\left(\beta_{11}\left(a+\frac{bd}{2}\right)+\delta P_1-D\right)dt_k\right\} \\ +\sum_{p=2}^{U-2}\left\{p\beta_{11}+\frac{p(p-1)}{2}g\right\}\left(a+\frac{bd}{2}\right)c_ft_k \\ -(U-3)(P_2-\delta P_3)c_ft_k-Dc_ft_k+\left\{\begin{array}{c}\left(U\beta_{11}+\frac{U(U-1)}{2}g\right)\left(a+\frac{bt_k}{2}\right) \\ -D-(P_2-\delta P_3)\end{array}\right. \\ +(P_2-\delta P_3)(M-U)\left(c_f-t_k\right)c_f\end{array}\right] \quad (19)$$

Carrying cost for non-usable products for i[th] cycle $(M+1\le i\le n)$ is as follows,

$$CN_i=\widetilde{R_2}\int_{d+(U-1)c_f}^{d+(M-1)c_f}S_{nu}(t)dt=\widetilde{R_2}\left\{\frac{(P_2-\delta P_3)}{2}(M-U)^2\left(c_f\right)^2\right\} \quad (20)$$

4.2.1. Total purchasing cost for used products

The expense of purchasing the used products is given by,Type equation here.

$$PC=\widetilde{R_4}\left[\sum_{i=2}^{U-1}\left\{i\beta_{11}+\frac{i(i-1)}{2}g\right\}\left(a+bc_f\right)+\left\{U\beta_{11}+\frac{U(U-1)}{2}g\right\}\left(a+bt_k\right)\right] \quad (21)$$

4.2.2. Total manufacturing cost with carbon emission tax for $(M-1)$cycle

Maintaining carbon emissions while enhancing the quality of the product in manufacturing is critical for any sustainable management strategy. Because of increased understanding of the demand to protect the natural world and legislation governing carbon emissions, manufacturers now consider carbon emissions to be one of the most important variables influencing production decisions. The overall manufacturing expense with carbon emission tax is provided by,

$$ME=(M-1)\widetilde{R_5}P_3O_m+(M-1)R_6P_3O_m+R_6P_1d_m \quad (22)$$

4.2.3. Total re-manufacturing cost with carbon emission tax

The overall re-manufacturing cost with carbon emission tax is as follows:

$$RM=(T-d)\widetilde{R_7}P_2+(T-d)R_6P_2 \quad (23)$$

4.2.4. Total screening cost for the whole system

The screening procedure is critical to maintaining the industry's reputation while also optimizing the system's economic income. Hence, the screening process is required to discover faulty goods, which must then be removed from the process for remanufacturing or disposal. The screening cost for the whole system is estimated by,

$$SC=(M-1)\widetilde{R_8}P_3O_m+\widetilde{R_8}P_1d_m \quad (24)$$

Total profit of the system is as follows,

$$\widetilde{TP}\left(U,c_f\right)=\left\{\begin{array}{c}Sales\ revenue-Total\ manufacturing\ cost\ with\ carbon\ emission \\ cost-Total\ remanufacturing\ cost\ with\ carbon\ emission\ cost \\ -Total\ purchasing\ cost-Total\ carrying\ cost \\ Total\ ordering\ cost-Total\ screening\ cost\end{array}\right\} \quad (25)$$

5. Optimality computational criteria

To achieve the optimum profit value per unit of $\widetilde{TP}\left(U,c_f\right)$ with respect to U and c_f.

The following steps are used:

Step 1: The 1st order derivative of $\widetilde{TP}(U, c_f)$is took w. r. t. continuous variable U and c_f.

$$\frac{\partial \widetilde{TP}(U,c_f)}{\partial U} = 0 \; \& \; \frac{\partial \widetilde{TP}(U,c_f)}{\partial c_f} = 0$$

Step 2: The equation from step 1 must be solved simultaneously in order to reach an optimal solution.

Step 3: Take 2nd order of $\widetilde{TP}(U, c_f)$ satisfying the condition

$$A_H = \begin{bmatrix} \frac{\partial^2 \widetilde{TP}(U,c_f)}{\partial U^2} & \frac{\partial^2 \widetilde{TP}(U,c_f)}{\partial c_f \partial U} \\ \frac{\partial^2 \widetilde{TP}(U,c_f)}{\partial U \partial c_f} & \frac{\partial^2 \widetilde{TP}(U,c_f)}{\partial (c_f)^2} \end{bmatrix}$$

$$\frac{\partial^2 \widetilde{TP}(U,c_f)}{\partial U^2}\frac{\partial^2 \widetilde{TP}(U,c_f)}{\partial (c_f)^2} - \left(\frac{\partial^2 \widetilde{TP}(U,c_f)}{\partial U \partial c_f}\right)^2 > 0$$

Numerical Analysis:

To validate the model, in this study, the manufacturer acquires used items from the market, and proceeds to recover and remanufacture after that some of the components and parts which cannot recycle led are disposed of. Numerical data has been taken from (Roy et al., 2009).To demonstrate the developed model, the numerical values have been considered as follows: P_1 =50,P_2 =5, P_3 =50,$(\widetilde{R_{11}}, \widetilde{R_{12}}, \widetilde{R_{13}}, \widetilde{R_{14}})$ = (1,2,3,4)$(\widetilde{R_{21}}, \widetilde{R_{22}}, \widetilde{R_{23}}, \widetilde{R_{24}})$ = (0.2,0.3,0.4,0.5), $(\widetilde{R_{31}}, \widetilde{R_{32}}, \widetilde{R_{33}}, \widetilde{R_{34}})$ = (5,10,15,20),$(\widetilde{R_{41}}, \widetilde{R_{42}}, \widetilde{R_{43}}, \widetilde{R_{44}})$ = (3,5,6,7), $(\widetilde{R_{51}}, \widetilde{R_{52}}, \widetilde{R_{53}}, \widetilde{R_{54}})$ = (10,15,20,25), R_6 =1, $(\widetilde{R_{71}}, \widetilde{R_{72}}, \widetilde{R_{73}}, \widetilde{R_{74}})$ = (1,2,3,4), R_8 =2, d =10, d_m =0.4, β_{11} =0.125, S =30, D =3.5, δ =0.05, T =8, U =4, a =12, b =0.3, g =0.02, J_1 =0.1, J_2 =10.

Result

The optimum results for U and c_f as well as the total profit from equation (25) have been evaluated. The results are presented in Table 1.

Table 1: Optimum results of overall profit, U J and c_f.

Overall Profit	U	c_f(Days)
15.2463	1.9643	0.62351

Sensitivity analysis:

This analysis is needed to understand the behavior of the parameter throughout the model acquiring the right sustainable manufacturing management solutions.

- When the amount of carbon emission tax raises, then the total profit of the system reduces and U i.e., the number of cycles in which serviceable goods have purchased, are decreases.
- If the value of Screening cost increases, then the total profit of the system decreases and U i.e., the number of cycles in which serviceable goods have purchased, are decrease.
- When both, carbon emission tax and Screening cost are increase, then $\boldsymbol{c_f}$ i.e., the duration of other cycles increases. Table 3 shows that the carbon emission and screening costs are both highly effective in terms of the system's total profit.

Table 2: Sensitivity analysis w. r. t. the parameters of the mentioned example.

Parameters	Change in %	TP	U	c_f
	-50%	15.3264	1.85008	0.63585
	-25%	15.0769	1.84999	0.62414
R_6	0	14.7594	1.85791	0.63926
	25%	14.7409	1.86782	0.64612
	50%	14.5024	1.84873	0.64421
	-50%	17.4272	1.85015	0.64587
	-25%	16.0828	1.85103	0.64501
R_8	0	14.7594	1.86871	0.6434
	25%	13.735	1.86879	0.64526
	50%	12.2806	1.87876	0.64538

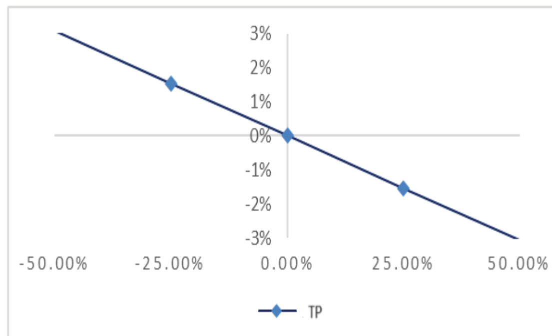

Fig 3. Effect of carbon emission cost on Total profit.

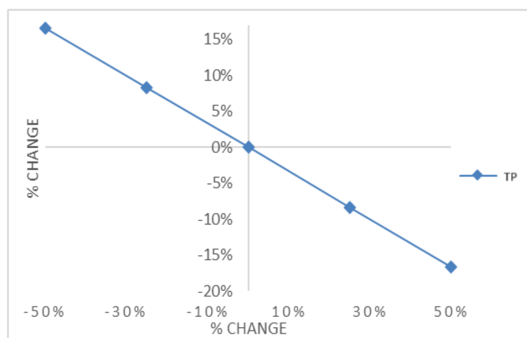

Fig 4. Effect of Screening cost on Total profit

8. Conclusion

The worth of resources in a sustainable society can be reused, refurbished, recycled and remanufactured again without diminishing their characteristics. Furthermore, this study showed that recycling is among the most important ways to sustainable manufacturing that emphasizes contamination and the conservation of the world's resources. A mathematical framework for sustainable inventory has been developed in which the linear demand is satisfied by manufactured and remanufactured items. The optimal result demonstrates that if the value of carbon emission and screening costs decreases, then the system's overall profit improves, which is an apparent reason. This represents a departure from traditional reverse logistics systems, which primarily focused on reusing materials to protect the environment. By promoting remanufacturing, this approach not only supports environmental sustainability but also offers significant benefits for industrial applications. It enhances both ecological and economic performance, providing a more comprehensive and advantageous strategy for modern industry practices. This dual focus on profitability and environmental responsibility presents a compelling case for integrating sustainable practices into industrial operations. In the real world, electronic gadgets are reconditioned and considered as viable things on the market. The manufacturing/remanufacturing idea may be used in the business scheme of electronic devices on the marketplace for any such items. Therefore, corporate managers, academics, and social organizers simply indicated as electronic parts manufacturers to take advantage of remanufacturing. We recommend that companies employ the screening-level evaluation discussed here, ensuring that they do not neglect significant sources of environmental consequences throughout their supply chains. Such data can assist businesses in pursuing carbon and ecological emission reduction efforts not only inside their operations, but also throughout their supply chain.

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